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Group and Multifuzzy Sets

[1*] Jency John, [2] K.V Thomas,[3] Anu V.
[1] (Research Scholar, Bharata Mata College,Thrikkakara)
Department of Mathematics,Sree Narayana College, Kannur, Kerala.
[2] Research Supervisor, Department of Mathematics,
Bharata Mata College, Thrikkakara, Kerala.
[3] Department of Mathematics, St. Peter's College, Kolenchery, Kerala.
*E-mail : mail2jency2@gmail.com

Abstract

In this paper we introduced p - binary operation to a special type fuzzy set called multifuzzy set. The tabular representation of p - binary operation is also incorporated. Using this p - binary operation, we defined p - Multifuzzy group ($pMFG$) mainly into three different ways : type1 $pMFG$, type2 $pMFG$ and type3 $pMFG$. In this direction we discuss some examples and properties of these $pMFG$s.

Keywords: Fuzzy Set, Group, Binary Operation, p-Binary Operation, Multifuzzy Set.

1 Introduction

A set is a "well defined "collection of objects.That means in a set (crisp) elements must be distinct and definite. If we avoid the restriction distinct we get multiset [1] and if we avoid the restriction definite we get fuzzy set [2] . Fuzzy multiset [3] is a generalization of fuzzy set, since every fuzzy set is a fuzzy multiset and it is also a generalization of multiset.

In 1986 Yager introduced the concept of fuzzy multisets. In fuzzy multisets an element of a set X may occur more than once with possibly the same or different membership values. For example, let $X = \{a, b, c\}$.

$A = \{(a, 0.6), (b, 0.2), (a, 0.8), (a, 0.6), (c, 0.8), (b, 0.1)\}$ is a fuzzy multiset of X.

The collection of all finite fuzzy multisets of X is denoted by $FM(X)$. For $x \in X$, the membership sequence is defined to be the decreasingly-ordered sequence of the elements in $count_A(x)$. It is denoted by $(\mu_A^1(x), \mu_A^2(x), ...\mu_A^p(x))$ where $\mu_A^1(x) \geqslant \mu_A^2(x) \geqslant ... \geqslant \mu_A^p(x)$.

In fuzzy multisets one can append any number of zeros at the right end of a finite sequence of the membership values of x. It does not make any difference in the occurrence of an element x. But in some situations this approach may be not good,that means there may be a chance that some members are present in certain situation with zero membership degree and some other members are absent there also. This is the case in Gregor Mendel's experiment, when Mendel put pollen from tall plants into the flowers of short plants, the seeds produced in F_1 generation were all tall plants. But the short trait was not lost.So we have to consider the short trait present in F_1 generation with zero membership degree. Self pollinating F_1 plants produced $1/4$ short and $3/4$ tall plants.Thus in F_2 generation the short trait was present with some membership degree. For another example observe a patient's symptoms continuously for 5 days for the purpose of medical diagnosis. The patient may have one particular symptom on day1,day2,day4 and day5 . Then we have to consider these days with non-zero membership degree corresponding to the ceverness of the symptom while day three with membership degree zero. So simply giving zero membership degree is not good. Here we consider a fuzzy multiset with zero membership degree, where zero membership degree also has a meaning.

Suppose we want to design a brochure with a given set of colour's. For this we consider the set with given colours as elements . The membership degree is given on the basis of intensity of colours. In this situation one colour with same intensity has no meaning. Hence in our Fuzzy multisets one element with same membership degree is not repeated. This type of fuzzy sets are called hesistant fuzzy set by author in [4]. We prefer to call this restricted fuzzy multiset as "multifuzzy set ".

We call "multifuzzy set" for this restricted fuzzy multisets in the sense that it contains multi fuzzymembership values for an element. But because of its two restrictions from fuzzy multisets and unimportance of ordering, this is not the same as multi-fuzzy set defined by Dr.Sabu Sebastian and Dr.T.V. Ramakrishnan in [5]. The collection of all multifuzzy sets (a fuzzy multiset in which zero membership degree also has a meaning and an element with same membership degree is not repeated) defined on a set X is denoted by $MF0(X)$ and an element of $MF0(X)$ is denoted by $MF_0(X)$.

We define a p - binary operation $*$ on $MF_0(X)$ as follows

1.1 p - Binary operation

Definition 1.1. A p-binary operation $*$ on $MF_0(X)$ is function from $MF_0(X)$ X $MF_0(X)$ to $MF_0(X)$, here $*$ is a pair $(.,+)$, where . is a binary operation (denoted by multiplication) defined on X and $+$ is a binary operation (denoted by addition) defined on the set M of membership values.

Example 1:

Let $X = \{0,1\}$ and $MF_0(X) = \{(0,0),(0,.5),(0,1),(1,0),(1,.5),(1,1)\}$. Then

i) $*_1 = (+_2, +_\odot)$ is a p-binary operation on $MF_0(X)$,where $x +_2 y =$ the remainder when $x + y$ is divided by 2 ; $x, y \in X$ and

$$a +_\odot b = \begin{cases} 0 & when\ a+b = 2\ or\ 0 \\ 1 & when\ a+b = 1 \\ the\ decimal\ part\ of\ a+b & otherwise. \end{cases}$$

where $a, b \in M$.

ii) If we define $*_2 = (+_2, min)$ then $*_2$ is a p - binary operation, where min is equal to the minimum of the ordered pair.

If we take underlying crisp set from $MF_0(X)$ (ie, if we take X) then p - binary operation is an extension (to $MF_0(X)$) of its binary operation.
$*$ is a commutative p - binary operation if it satisfy commutative property. Obviously $*$ is commutative if and only if both the binary operation . and + are commutative. The p - binary operations defined in Example 1 are commutative p-binary operations.

Example 2:

The p - binary operation defined by $(x, a_i) * (y, b_j) = (x +_2 y, a_i)$ is a non-commutative p - binary operation defined on
$MF_0(X) = \{(0,0),(0,1),(1,0),(1,1)\}$.
In the same way a p - binary operation $*$ on $MF_0(X)$ is associative if it satisfies the associative property. Obviously $*$ is associative if and only if both the binary operations . and + are associative.
In the crisp set a binary operation defined on a finite set can be represented by tables. Same way a p - binary operation defined on $MF_0(X)$ also can be

represented by using table. But the traditional representation is complicated in this case. Hence we represented in the following way. For a finite $MF_0(X)$, the elements of X which are involved in $MF_0(X)$ listed with its membership degree as subscripts, across the top as heads of columns and at the left side as heads of rows. We have to consider the body of the representation as a matrix form. Let the entries of $X = \{a_i / i \in \{1, 2, 3...n\}\}$ and the membership degree of a_i is denoted by $b_{i1}, b_{i2}, b_{i3}...b_{ik_i}$, where k_i is any positive integer. Then the $(i, j)th$ entry of the matrix is $a_i.a_j$ with a suffix matrix whose $(p, q)th$ entry is $b_{ip} + b_{jq}$.

Example 3:

Let $X = \{a, b\}$ and $MF_0(X) = \{(a, 0), (a, 1), (b, .5)\}$. Then $* = (., +_{\odot})$ is a p - binary operation, if we define . as $a.a = a, a.b = b, b.a = b$ and $b.b = a$ and $, +_{\odot}$ is same as we defined in Example 1.

$(+, .)$	$a_{0,1}$	$b_{.5}$
$a_{0,1}$	$a\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$b\begin{bmatrix} .5 \\ .5 \end{bmatrix}$
$b_{.5}$	$b\begin{bmatrix} .5 & .5 \end{bmatrix}$	$a\begin{bmatrix} 1 \end{bmatrix}$

This representation is more convenient for checking the commutativity of p - binary operation. A p - binary operation is commutative if and only if the representation satisfies the following conditions.

1) The entries in the main matrix (the matrix corresponding to the binary operation) are symmetric with respect to the diagonal that starts at the upper left corner of the table and terminates at the lower right corner.

2) The suffix matrix of the (i,j)th entry of the main matrix is the transpose of the suffix matrix of the (j,i)th entry of the main matrix. From the

representation, it is clear that the p - binary operation defined in Example 3 is commutative. The table representation of Example 2 is

$(+,.)$	$0_{0,1}$	$1_{0,1}$
$0_{0,1}$	${}^{0}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	${}^{1}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$
$1_{0,1}$	${}^{1}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	${}^{0}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

Here the representation satisfy the first condition but it does not satisfies the second conditions. So it is a non-commutative binary operation.

1.2 p - Multifuzzy group($pMFG$)

Fuzzy set is a generalization of crisp set. Here we generalize definition of group into Multifuzzy sets. We define three basic p- multifuzzy groups.

Definition 1.2. Let $(G, .)$ be a group. Then a multifuzzy set $MF_0(G)$ together with a p - binary operation $* = (., +)$ is called a p - multifuzzy group if it satisfies the following axioms:

$pMFG_1$: $+$ is associative
ie,$(a + b) + c = a + (b + c)$ for all membership values in $MF_0(G)$

$pMFG_2$: The identity element e of G must exist in $MF_0(G)$ with membership degree one.

$pMFG_3$: For each x with membership degree a_i in $MF_0(G)$ the inverse of x (ie, x^{-1}) in G exist in $MF_0(G)$ with membership degree $1 - a_i$.

This type of p - multifuzzy group is called type1 p - multifuzzy group.

If $pMFG_3$ is replaced by $pMFG_{3'}$ where
$pMFG_{3'}$: For each x with membership degree a_i in $MF_0(G)$ the inverse of x (ie, x^{-1}) in G exist in $MF_0(G)$ with same (ie, a_i) membership degree.
then this type p - multifuzzy group is called type2 p - multifuzzy group.
If $pMFG_3$ is replaced by $pMFG_{3''}$ where
$pMFG_{3''}$: For each $(x, a_i) \in MF_0(G)$, there exist an inverse element $(x, 1 - a_i) \in MF_0(G)$.
then p - multifuzzy group is called type3 p - multifuzzy group.

Note that we are not insisting that $(x, a_i) * (x, a_i)^{-1} = (e, 1)$. We have to define a special type3 p - MFG by relaxing the condition $(G, .)$ be a group to $(G, .)$ be a monoid. Three basic p - multifuzzy groups are defined here. Using different fuzzy complements we can define different p - multifuzzy groups.

Example 4 : Certain p -multifuzzy group in Z_2

a : $MF_0^1(Z_2) = \{(0,0), (0,1), (1,0), (1,1)\}$ are type1,type2,type3 p - multifuzzy groups with the operation $* = (+_2, , +_\odot)$ or any other associative p - binary operations.

b : $MF_0^2(Z_2) = \{(0,0), (0,1), (1,.5)\}$ are type1,type2,type3 p - multifuzzy groups with the operation $* = (+_2, , +_\odot)$ or any other associative p - binary operations.

c : $MF_0^3(Z_2) = \{(0,0), (0,1), (1,.2)\}$. is a type2 $pMFG$. But it is not type1 and type3 $pMFG$, since inverse of $(1, .2)$ does not exist.

d : Example of a $pMFG$ with each element has infinitely many membership degree.

$MF_0^4(Z_2) = \{0 \setminus [0, .1) \cup (.9, 1], 1 \setminus [0, .1) \cup (.9, 1]\}$ is a pMG of type1,type2 and type3 with the p-binary operation $* = (+_2, +_{.1})$

In $(Z_2, +_2)$ type1 $pMFG$ and type3 $pMFG$ are the same,since in Z_2 each element is its own inverse. Similarly we can define $pMFG$ in Z_n for all n. More generally we can define infinitely many $pMFG$s in any group G.

Example 5 :

Consider an infinite group Z,

$MF_0(Z) = \{... - 1 \setminus [0, .2) \cup (.8, 1], 0 \setminus [0, .2) \cup (.8, 1], 1 \setminus [0, .2) \cup (.8, 1], ...\}$ is a pMG of type1,type2 and type3 with the binary operation $* = (+, +_{.2})$

Example 6 :

Let $D_3 = \{\rho_0, \rho_1, \rho_2, \mu_1, \mu_2, \mu_3\}$, the symmetric group of an equilateral triangle. Consider the subgroup of $G = \{\rho_0, \rho_1, \rho_2\}$ of D_3. This subgroup is itself a group. Let

$MF_0(G) = \{\rho_0 \setminus \{0, .2, .4, .6, .8, 1\}, \rho_1 \setminus \{0, .2, .4, .6, .8, 1\}, \rho_3 \setminus \{0, .2, .4, .6, .8, 1\}\}$.

Then $MF_0(G)$ is a $pMFG$ of type1,type2 and type3 under the binary operation $* = (., +_1)$

1.3 Properties

1 . The identity element $(e, 1)$ is unique in $pMFG$ (Since e is unique in G). e may have different membership values.

2 . For all the three type $pMFG$s, every $(x, a_i) \in MF_0(G)$ there exists a unique inverse. We denote the inverse of (x, a_i) by $(x, a_i)'$ or by $(x, a_i)^{-1}$.

3 . For all the 3 type $pMFG$s

$((x, a_i)^{-1})^{-1} = (x, a_i)$ for all (x, a_i)

4 . In type2 $pMFG$, $[(x,a_i)*(y,b_j)]^{-1}=(y,b_j)^{-1}*(x,a_i)^{-1}$, for all (x,a_i) and (y,b_j).But in type1 and type3 $pMFG$s
$[(x,a_i)*(y,b_j)]^{-1}\neq(y,b_j)^{-1}*(x,a_i)^{-1}$.

Property four is illustrated by the following example.

Example 7:

$MF_0(Z_2)=$
$\{(0,0),(0,.2),(0,.4),(0,.6),(0,.8),(0,1),(1,0),(1,.2),(1,.4),(1,.6),(1,.8),(1,1)\}$
is $pMFG$s of type1,type2 and type3. with the binary operation $(+_2,min)$.
In type1 and type3 $pMFG$s :
$[(1,.2)*(0,.6)]^{-1}=(1,.2)^{-1}=(1,.8)$ but
$(0.6)^{-1}*(1,.2)^{-1}=(0,.4)*(1,.8)=(1,.4)$.

1.4 Conclusion

In this paper we introduced the concept of p - binary operation. Building upon this foundational definition, we subsequently defined p - multifuzzy group mainly in three different ways. Certain examples and properties of these also discussed.

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